B.Sc. Semester III (Honours) Examination, 2018-19

PHYSICS

Course ID : 32411 **Course Code : SHPHS-301C-5(T)**

Course Title : Mathematical Physics II

Time: 1 Hour 15 Minutes **Full Marks: 25**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Section–I

1. Answer *any five* questions: 1×5=5

(a) Complex number $Z = 1 + \sqrt{3}\,\hat{\imath}$, write it in polar form.

(b) State Cauchy's integral theorem.

(c) What is cyclic co-ordinate?

(d) If λ be an eigenvalue of a matrix A (non-zero matrix), show that λ^{-1} is an eigenvalue of the matrix A^{-1}.

(e) What is the nature of singular point for a complex function $f(z) = \frac{\sin z}{z}$?

(f) Calculate the probability of obtaining 4 heads in 6 tosses using an unbiased coin.

(g) Define a linear operator.

(h) Write down Lagrangian equation for a simple pendulum.

Section–II

Answer *any two* questions: 5×2=10

2. (a) Prove that $u = e^{-x}\,(x\sin y - y\cos y)$ is harmonic.

(b) Find v such that $f(z) = (u + iv)$ is analytic. 2+3=5

3. Find the eigenvalues and eigenvectors of the given matrix.

$$A = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$$ 2+3=5

4. Show that shortest distance between two points is always a straight line. 5

5. Show that Dirac delta function can be represented as a limit of a Gaussian function and rectangular function. 5

Section–III

Answer *any one* question: 10×1=10

6. (a) Show that eigenvalues of a Hermitian matrix are real.

(b) What is similarity transformation? Diagonalize the matrix $A = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$ through similarity transformation.

(c) Prove that the matrix $\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is unitary matrix. 3+(1+3)+3=10

7. (a) Evaluate $\int_0^\infty \frac{dx}{x^6+1}$.

(b) Find the residue of the complex function $f(z) = \frac{e^z}{z^4}$.

(c) Evaluate $\int_{1-i}^{2+i} (2x + iy + 1)\, dz$ along the straight line joining $(1 - i)$ and $(2 + i)$. 5+2+3=10

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